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Supersymmetry Transformation of Quantum Fields

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Abstract

In the Wess-Zumino gauge, supersymmetry transformations become non-linear and are usually incorporated together with BRS transformations in the form of Slavnov-Taylor identities, such that they appear at first sight to be even non-local. Furthermore, the gauge fixing term breaks supersymmetry. In the present paper, we clarify in which sense supersymmetry is still a symmetry of the system and how it is realized on the level of quantum fields.

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1 Introduction

Electroweak processes in todays particle physics are very successfully described by the standard model [1]. The precision of the LEP II experiments will require two loop corrections [2], which in turn will be calculable because the algebraic method of renormalization has been extended to the standard model [3]. This method permits an unabiguous construction of the model via the Slavnov-Taylor (ST) identitiy and a rigid and a local gauge Ward identity (WI) in a way independent from the applied renormalization scheme. At present the only viable alternative to the SM is a supersymmetric extension thereof, e.g. a minimal one (MSSM). Here the status of loop corrections is much less satisfactory because the effect of the renormalized supersymmetry WI has not yet been systematically taken into account. A first step into this direction has been undertaken in [4] for the susy extension of QED. It applies techniques developped in [5, 6] and gives clear prescriptions on how to establish and use the susy WI. This is non-trivial because in the chosen Wess-Zumino gauge [7] the susy transformations are non-linear and – as is shown in [4] – also non-local. The question then arises in which sense susy is still a symmetry of the system. Clearly, it is realized on the Green functions but is it realized on the local fields, on the state space? Does there exist a susy charge which commutes with the S-matrix? In the present paper we wish to answer these questions for the Wess-Zumino model and then for SQED, basing our analysis on the results of [4].

2 The Wess-Zumino model without auxiliary fields

The model comprises a complex scalar and a (Weyl) spinor field together with their conjugates, forming the classical action

$$\Gamma_{\rm WZ} = \int d^4x \left\{ \partial^\mu\phi\partial_\mu\bar\phi - m^2\phi\bar\phi + \tfrac{i}{2}\psi\sigma^\mu\partial_\mu\bar\psi + \tfrac{1}{4}m(\psi\psi + \bar\psi\bar\psi) \right.$$
$$\left. - \tfrac{1}{16}g^2\phi^2\bar\phi^2 + \tfrac{g}{8}\psi\psi\phi + \tfrac{g}{8}\bar\psi\bar\psi\bar\phi - \tfrac{1}{4}mg\phi\bar\phi(\phi+\bar\phi) \right\} \qquad (2.1)$$

It has one common mass for all fields and only one coupling. This is due to supersymmetry and parity which leave invariant this action. Without auxiliary fields the susy transormations are non-linear and their algebra closes only on the mass shell - a problem for renormalization. It has been solved in [8] by introducing external fields coupled to the non-linear variations and formulating the Ward identity (WI) for susy as a Γ-bilinear equation. Crucial is the fact that in $\Gamma_{\rm eff}$ appears an expression bilinear in the external fields. Meanwhile this method has been greatly refined [5, 6] and systematized, hence we shall present the relevant results in this form.

2.1 BRS transformations, ST identity

The systematic procedure for treating susy without auxiliary fields consists in promoting the parameters of the transformations to constant ghosts. Those carry a Grassmann number which is always opposite to its statistics: the ghosts of susy (ϵ^α, $\bar\epsilon_{\dot\alpha}$) commute, the translations (ω_μ) anticommute. In order to have nilpotent transformations the ghosts too have to transform. With the assignment

$$\mathbf{s}\,\phi = \varepsilon^\alpha\psi_\alpha - \mathrm{i}\omega^\mu\partial_\mu\phi \tag{2.2}$$

$$\mathbf{s}\,\psi^\beta = \tfrac{g}{2}\varepsilon^\beta\bar\phi^2 + 2\varepsilon^\beta m\bar\phi - 2\mathrm{i}\bar\varepsilon_{\dot\alpha}\sigma^{\mu\beta\dot\alpha}\partial_\mu\phi - \mathrm{i}\omega^\mu\partial_\mu\psi^\beta \tag{2.3}$$

$$\mathbf{s}\,\bar\phi = -\bar\varepsilon_{\dot\alpha}\bar\psi^{\dot\alpha} - \mathrm{i}\omega^\mu\partial_\mu\bar\phi \tag{2.4}$$

$$\mathbf{s}\,\bar\psi^{\dot\beta} = -\tfrac{g}{2}\bar\varepsilon^{\dot\beta}\phi^2 - 2\varepsilon^{\dot\beta}m\phi - 2\mathrm{i}\varepsilon_\alpha\sigma^{\mu\alpha\dot\beta}\partial_\mu\bar\phi - \mathrm{i}\omega^\mu\partial_\mu\bar\psi^{\dot\alpha} \tag{2.5}$$

$$\mathbf{s}\,\varepsilon^\alpha = 0 = s\bar\varepsilon^{\dot\alpha} \tag{2.6}$$

$$\mathbf{s}\,\omega^\mu = 2\varepsilon\sigma^\mu\bar\varepsilon \tag{2.7}$$

one finds

$$\mathbf{s}^{\,2}\phi = 0 = \mathbf{s}^{\,2}\bar\phi \tag{2.8}$$

$$\mathbf{s}^{\,2}\psi^\beta = -4\varepsilon^\beta\bar\varepsilon^{\dot\beta}\frac{\delta\Gamma_{\mathrm{WZ}}}{\delta\bar\psi^{\dot\beta}} \tag{2.9}$$

$$\mathbf{s}^{\,2}\bar\psi^{\dot\beta} = 4\varepsilon^\beta\bar\varepsilon^{\dot\beta}\frac{\delta\Gamma_{\mathrm{WZ}}}{\delta\psi^\beta} \tag{2.10}$$

In order to obtain off-shell closure one introduces external fields coupled to the non-linear variations and a term bilinear in these external fields

$$\Gamma_{\mathrm{ext}} = \int \left\{ Y^\alpha\, s\psi_\alpha + \bar Y_{\dot\alpha}\, s\bar\psi^{\dot\alpha} + 4\varepsilon^\alpha Y_\alpha\, \bar\varepsilon_{\dot\alpha}\bar Y^{\dot\alpha} \right\} \tag{2.11}$$

$$\Gamma_{\mathrm{cl}} = \Gamma_{\mathrm{WZ}} + \Gamma_{\mathrm{ext}} \tag{2.12}$$

The symmetry of the model can now be expressed as a ST identity

$$\mathscr{S}(\Gamma) \equiv \int \left\{ s\phi\frac{\delta\Gamma}{\delta\phi} + s\bar\phi\frac{\delta\Gamma}{\delta\bar\phi} + \frac{\delta\Gamma}{\delta Y_\alpha}\frac{\delta\Gamma}{\delta\psi^\alpha} + \frac{\delta\Gamma}{\delta\bar Y^{\dot\alpha}}\frac{\delta\Gamma}{\delta\bar\psi_{\dot\alpha}} \right\} + s\omega^\mu\frac{\partial\Gamma}{\partial\omega^\mu} \;=\; 0\,. \tag{2.13}$$

It holds for $\Gamma = \Gamma_{\mathrm{cl}}$. One observes furthermore that the linearized operator

$$\mathscr{S}_\Gamma \equiv \int \left(\mathbf{s}\,\phi\frac{\delta}{\delta\phi} + \mathbf{s}\,\bar\phi\frac{\delta}{\delta\bar\phi} + \frac{\delta\Gamma}{\delta Y_\alpha}\frac{\delta}{\delta\psi^\alpha} + \frac{\delta\Gamma}{\delta\psi^\alpha}\frac{\delta}{\delta Y_\alpha} + \frac{\delta\Gamma}{\delta\bar Y^{\dot\alpha}}\frac{\delta}{\delta\bar Y_{\dot\alpha}} + \frac{\delta\Gamma}{\delta\bar Y^{\dot\alpha}}\frac{\delta}{\delta\bar Y_{\dot\alpha}} \right) + \mathbf{s}\,\omega^\mu\frac{\partial}{\partial\omega^\mu} \tag{2.14}$$

satisfies

$$\mathscr{S}_\Gamma^2 = 0 \tag{2.15}$$

$$\mathscr{S}_\Gamma\mathbf{s}\,\phi = 0 \tag{2.16}$$

$$\mathscr{S}_\Gamma\mathscr{S}(\Gamma) = 0 \tag{2.17}$$

for $\Gamma = \Gamma_{\mathrm{cl}}$. Crucial for the validity of $(\mathscr{S}_\Gamma^2 = 0)$ is the presence of the $Y\bar{Y}$-term in (2.11). It clearly contributes those eq. of motion terms which guarantee on the functional level nilpotency as opposed to (2.9), (2.10) where – on the level of elementary fields – it does not hold. Eqn. (2.16) will serve as a consistency condition for constraining potential non-symmetric higher order corrections to the ST identity (2.13).

The renormalization is now straightforward [8] and yields the ST identity (2.13) if one imposes in addition

$$\frac{\partial\Gamma}{\partial\omega^\mu} = \frac{\partial\Gamma_{\mathrm{cl}}}{\partial\omega^\mu} \tag{2.18}$$

which is possible, and recursively

$$(\mathscr{S}_\Gamma \mathbf{s}\,\phi) = 0\,. \tag{2.19}$$

(2.18) and (2.19) imply

$$\frac{\partial\Gamma}{\partial\omega^\mu} = -\mathrm{i}\int \left(Y^\alpha \partial_\mu \psi_\alpha + \bar{Y}_{\dot\alpha}\partial_\mu \bar{\psi}^{\dot\alpha}\right) \tag{2.20}$$

$$\varepsilon^\alpha \frac{\partial\Gamma}{\partial Y^\alpha} = -\mathrm{i}\varepsilon^\alpha \omega^\mu \partial_\mu \psi_\alpha + 2\mathrm{i}\varepsilon\sigma^\mu\bar{\varepsilon}\partial_\mu\phi \tag{2.21}$$

In order to prepare for the subsequent discussion we go via Legendre transformation over to the connected Green functions

$$-j = \frac{\delta\Gamma}{\delta\phi}\,, \qquad \frac{\delta Z_c}{\delta Y^\alpha} = \frac{\delta\Gamma}{\delta Y^\alpha}\,, \qquad -\eta_\alpha = \frac{\delta\Gamma}{\delta\psi^\alpha} \tag{2.22}$$

$$Z_c = \Gamma + \int \left(j\phi + \bar{j}\bar{\phi} + \eta_\alpha\psi^\alpha + \bar{\eta}_{\dot\alpha}\bar{\psi}^{\dot\alpha}\right) \tag{2.23}$$

and then to the general Green functions

$$Z = \mathrm{e}^{\mathrm{i}Z_c} \tag{2.24}$$

The validity of the ST identity (2.13) entails the existence of a conserved current for a *local* generalized BRS transformation

$$\mathscr{S}_{\mathrm{loc}}(y) Z = [\partial^\mu J_\mu(y)]\cdot Z \tag{2.25}$$

where

$$\begin{aligned}\mathscr{S}_{\mathrm{loc}} \equiv\ & \mathrm{i}j\varepsilon^\alpha \frac{\delta}{\delta\eta^\alpha} - \omega^\mu\partial_\mu j\frac{\delta}{\delta j} - \mathrm{i}\bar{j}\bar{\varepsilon}_{\dot\alpha}\frac{\delta}{\delta\bar{\eta}_{\dot\alpha}} - \omega^\mu\partial_\mu\bar{j}\frac{\delta}{\delta\bar{j}} - \mathrm{i}\eta^\alpha\frac{\delta}{\delta Y^\alpha} - \mathrm{i}\bar{\eta}_{\dot\alpha}\frac{\delta}{\delta\bar{Y}_{\dot\alpha}} \\ & - 2\varepsilon\sigma^\mu\bar{\varepsilon}\left(Y^\alpha\partial_\mu\frac{\delta}{\delta\eta^\alpha} + \bar{Y}_{\dot\alpha}\partial_\mu\frac{\delta}{\delta\bar{\eta}_{\dot\alpha}}\right)\,. \end{aligned} \tag{2.26}$$

2.2 Supersymmetry transformations of functionals

The presence of constant ghosts in the theory permits the subdivision of functionals and sometimes also of their transformations into sectors of fixed ghost number – this is commonly called a "filtration" [5, 6]. We expand

$$\Gamma = \Gamma^{(0)} + \Gamma^{(1)} + \dots \tag{2.27}$$

according to the number of constant ghosts. Inserting this expansion into the ST identity the latter can first be rewritten as

$$\mathscr{S}(\Gamma) = \mathscr{S}(\Gamma^{(0)}) + \mathscr{S}_{\Gamma^{(0)}}\Gamma^{(1)} + O(\epsilon^2) \tag{2.28}$$

where

$$\mathscr{S}(\Gamma^{(0)}) = \int \left(\mathbf{s}\,\phi \frac{\delta\Gamma^{(0)}}{\delta\phi} + \frac{\delta\Gamma^{(0)}}{\delta Y}\frac{\delta\Gamma^{(0)}}{\delta\psi} \right) + c.c. \tag{2.29}$$

(2.28) is verified by observing that $\frac{\delta\Gamma^{(0)}}{\delta Y} = 0$. Using

$$\mathscr{S}_{\Gamma^{(0)}}\Gamma^{(1)} = \int \left(\frac{\delta\Gamma^{(1)}}{\delta Y}\frac{\delta\Gamma^{(0)}}{\delta\psi} + c.c. \right) + \mathbf{s}\,\omega^\mu \frac{\partial\Gamma}{\partial\omega^\mu} + O(\epsilon^2) \tag{2.30}$$

one can write

$$\mathscr{S}(\Gamma) = (\mathscr{S}_\Gamma)^{(1)}\Gamma^{(0)} + O(\epsilon^2)\,, \tag{2.31}$$

hence, since the ST identity holds order by order in the ghosts

$$\mathscr{S}^{(1)}_\Gamma\Gamma^{(0)} = 0\,. \tag{2.32}$$

We decompose

$$\mathscr{S}^{(1)}_\Gamma = \epsilon^\alpha W_\alpha + \bar\epsilon^{\dot\alpha}\bar W_{\dot\alpha} + \omega^\mu W_\mu - 2\epsilon\sigma^\mu\bar\epsilon\frac{\partial}{\partial\omega^\mu} \tag{2.33}$$

such that functional differential operators are defined by differentiation w.r.t. the ghosts

$$W_\alpha = \frac{\partial}{\partial\epsilon^\alpha}\mathscr{S}^{(1)}_\Gamma\Big|_{\epsilon=\omega=0} \tag{2.34}$$

$$\bar W_{\dot\alpha} = \frac{\partial}{\partial\bar\epsilon^{\dot\alpha}}\mathscr{S}^{(1)}_\Gamma\Big|_{\epsilon=\omega=0} \tag{2.35}$$

$$W_\mu = \frac{\partial}{\partial\omega^\mu}\mathscr{S}^{(1)}_\Gamma\Big|_{\epsilon=\omega=0} \tag{2.36}$$

The filtration of $\mathscr{S}_\Gamma$ contains besides $\mathscr{S}^{(1)}_\Gamma$

$$\mathscr{S}^{(0)}_\Gamma = \frac{\delta\Gamma^{(0)}}{\delta\psi^\alpha}\frac{\delta}{\delta Y_\alpha} + c.c. \tag{2.37}$$

$$\mathscr{S}^{(2)}_\Gamma = \frac{\delta\Gamma^{(2)}}{\delta Y_\alpha}\frac{\delta}{\delta\psi_\alpha} + c.c. \tag{2.38}$$

Next we exploit the fact that $\mathscr{S}_\Gamma$ is nilpotent on functionals $\mathscr{F}$ for which

$$\varepsilon^\alpha \frac{\partial \mathscr{F}}{\partial Y^\alpha} = -\mathrm{i}\varepsilon^\alpha \omega^\mu \partial_\mu \psi_\alpha + 2\mathrm{i}\varepsilon\sigma^\mu \bar{\varepsilon}\partial_\mu \phi \tag{2.39}$$

holds. This yields

$$\mathscr{S}_\Gamma^{(0)} \mathscr{S}_\Gamma^{(0)} = 0 \tag{2.40}$$

$$\{\mathscr{S}_\Gamma^{(0)}, \mathscr{S}_\Gamma^{(1)}\} = 0 \tag{2.41}$$

$$\mathscr{S}_\Gamma^{(1)} \mathscr{S}_\Gamma^{(1)} + \{\mathscr{S}_\Gamma^{(0)}, \mathscr{S}_\Gamma^{(2)}\} = 0 \tag{2.42}$$

Projecting out of $\mathscr{S}_\Gamma^{(1)} \mathscr{S}_\Gamma^{(1)}$ the $\epsilon\bar{\epsilon}$-part one finds

$$\mathscr{S}_\Gamma^{(1)} \mathscr{S}_\Gamma^{(1)}\Big|_{\epsilon\bar{\epsilon}} = \epsilon^\alpha \bar{\epsilon}^{\dot{\alpha}} \left(\{W_\alpha, \bar{W}_{\dot{\alpha}}\} - 2\sigma^\mu_{\alpha\dot{\alpha}} W_\mu\right) . \tag{2.43}$$

(2.42) with (2.43) shows that even on Y-independent functionals $\mathscr{F}$, the SUSY algebra closes only up to equation of motions terms $\frac{\delta \mathscr{F}}{\delta \psi}$.

2.3 Symmetry transformations on quantum fields

Vertex functions can essentially be understood as matrix elements of operators but are certainly not the ideal tool for revealing the underlying properties of the respective operators, hence we study general Green functions.

As the simplest example we identify the energy-momentum operator and its action on field operators. We therefore insert (2.21) into (2.13) and permit ω_μ, ε^α and $\bar{\varepsilon}_{\dot{\alpha}}$ to be local, $\omega_\mu = \omega_\mu(x)$ etc.,

$$\mathscr{S}_{\mathrm{loc}} Z = [\partial^\mu J_\mu + \partial^\mu \varepsilon^\alpha K_{\mu\alpha} + \partial^\mu \bar{\varepsilon}^{\dot{\alpha}} \bar{K}_{\mu\dot{\alpha}} + \partial^\mu \omega^\nu K_{\mu\nu}] \cdot Z\, . \tag{2.44}$$

Differentiating w.r.t. $\omega_\mu(z)$ and integrating over z yields the local WI

$$w_\mu \Gamma|_{\epsilon=\bar{\epsilon}=0, Y=\bar{Y}=0} \equiv \mathrm{i}\left(\partial_\mu \phi \frac{\delta}{\delta\phi} + \partial_\mu \bar{\phi}\frac{\delta}{\delta\bar{\phi}} + \partial_\mu \psi \frac{\delta}{\delta\psi} + \partial_\mu \bar{\psi}\frac{\delta}{\delta\bar{\psi}}\right)\Gamma\Big|_{\substack{\epsilon=\bar{\epsilon}=0\\ Y=\bar{Y}=0}} \tag{2.45}$$

$$= [\partial^\nu T_{\nu\mu}] \cdot \Gamma|_{\substack{\epsilon=\bar{\epsilon}=0\\ Y=\bar{Y}=0}} \ , \tag{2.46}$$

with $T_{\nu\mu} = \partial_{\omega^\nu} J_\mu$. ((2.20) taken at $Y = 0 = \bar{Y}$ guarantees that no other contributions depending on ω_μ show up.) Translated onto Z we obtain

$$\left(\partial_\mu j_\phi \frac{\delta}{\delta j_\phi} + \partial_\mu \eta \frac{\delta}{\delta \eta} + c.c.\right) Z\Big|_{Y=0} = -\mathrm{i}[\partial^\nu T_{\nu\mu}] \cdot Z \tag{2.47}$$

a local WI for the translations, with $T_{\nu\mu}$ being the energy-momentum tensor. By differentiating with respect to a suitable combination of sources we generate on the right

hand side the Green function $-\partial^\nu\langle \mathrm{T}(T_{\nu\mu}X)\rangle$, where X stands for an arbitrary number of elementary fields; on the left hand side there will always occur Green functions with one field argument less and a δ-function instead. Multiplying with inverse propagators and going on mass shell we obtain zero on the l.h.s. (as a consequence of the δ-functions) and on the r.h.s. the respective matrix elements of the operator $\partial^\nu T^{\mathrm{Op}}_{\nu\mu}$. Hence

$$\partial^\nu T^{\mathrm{Op}}_{\nu\mu} = 0\,, \tag{2.48}$$

the energy-momentum tensor is conserved.

Defining the energy-momentum operator by

$$P^{\mathrm{Op}}_\nu = \int \mathrm{d}^3x\, T^{\mathrm{Op}}_{0\nu}\,, \tag{2.49}$$

(2.48) means that P^{Op}_ν is time independent. This detailed presentation served as an illustration of the LSZ reduction technique which permits one to generate operator relations out of Green functions [10].

In order to obtain the transformation law of field operators we deduce from (2.47)

$$\partial^y_\nu \delta(y-x)\,\langle\varphi(y)\,X\rangle = -\mathrm{i}\partial^{y\mu}\langle \mathrm{T}(T_{\mu\nu}(y)\varphi(x)X)\rangle\,. \tag{2.50}$$

Reduction yields

$$\partial^y_\nu \delta(y-x)\varphi^{\mathrm{Op}}(y) = -\mathrm{i}\partial^{y\mu}\mathrm{T}(T_{\mu\nu}(y)\varphi(x))^{\mathrm{Op}}\,. \tag{2.51}$$

Integration over $\int \mathrm{d}^3y$ and $\int_{x^0-\varepsilon}^{x^0+\varepsilon}\mathrm{d}y^0$ leads to

$$\partial_\mu\varphi^{\mathrm{Op}}(x) = \mathrm{i}[P_\mu,\varphi(x)]^{\mathrm{Op}}\,,\qquad \text{for } \varphi=\phi,\bar\phi,\psi,\bar\psi \tag{2.52}$$

the well-known transformation law of a field operator under translations.

We learn from this result that the local version of the ST identity on Z (2.44) should also be most useful for deriving the susy current, charge and field transformation.

$$\begin{aligned}
\mathrm{i}\delta(y-z)j(z)\frac{\delta Z}{\delta\eta_\alpha(z)} &+ \mathrm{i}\eta^\beta\left[\frac{\delta\Gamma_{\text{eff}}}{\delta\epsilon^\alpha}\right]\cdot\left[\frac{\delta\Gamma_{\text{eff}}}{\delta Y^\beta}\right]\cdot Z + \eta^\beta\left[\frac{\delta^2\Gamma_{\text{eff}}}{\delta\epsilon^\alpha(z)\,\delta Y^\beta(y)}\right]\cdot Z\\
&+ \mathrm{i}\bar\eta^{\dot\beta}\left[\frac{\delta\Gamma_{\text{eff}}}{\delta\epsilon^\alpha}\right]\cdot\left[\frac{\delta\Gamma_{\text{eff}}}{\delta\bar Y^{\dot\beta}}\right]\cdot Z + \eta^\beta\left[\frac{\delta^2\Gamma_{\text{eff}}}{\delta\epsilon^\alpha(z)\,\delta\bar Y^{\dot\beta}(y)}\right]\cdot Z\\
=\left[\partial^\mu_y\frac{\delta J_\mu(y)}{\delta\varepsilon^\alpha(z)}\right]\cdot Z &+ \mathrm{i}[\partial^\mu J_\mu(y)]\cdot\left[\frac{\delta\Gamma_{\text{eff}}}{\delta\varepsilon^\alpha(z)}\right]\cdot Z + \partial^\mu_y\delta(y-z)\,[K_{\mu\alpha}(y)]\cdot Z
\end{aligned} \tag{2.53}$$

Since ε has ghost number one, it may occur only in combination with Y, hence no ϵ-dependent term survives at $Y=0$, and the double insertions in (2.53) are absent. One can safely integrate over z which yields

$$\mathrm{i}j(y)\frac{\delta Z}{\delta\eta_\alpha(y)} - \eta^\beta(y)\frac{\delta\partial_{\epsilon^\alpha}\Gamma_{\text{eff}}}{\delta Y^\beta(y)} + \mathrm{i}\bar\eta^{\dot\beta}(y)\frac{\delta\partial_{\epsilon^\alpha}\Gamma_{\text{eff}}}{\delta\bar Y^{\dot\beta}(y)} = [\partial^\mu J_{\mu\alpha}(y)]\cdot Z\,, \tag{2.54}$$

where $J_{\mu\alpha} \equiv \partial_{\varepsilon^\alpha} J_\mu$ represents the susy current. From here on one may just perform all steps as before for the translations and obtain

$$\partial^\mu J^{\mathrm{Op}}_{\mu\alpha} = 0\,, \tag{2.55}$$

the current conservation. With the charge

$$Q_\alpha \equiv -\int \mathrm{d}^3x\, J_{0\alpha}(x) \tag{2.56}$$

the field transformations follow,

$$\mathrm{i}[Q_\alpha, \phi]^{\mathrm{Op}} = \psi^{\mathrm{Op}}_\alpha \tag{2.57}$$

$$\mathrm{i}\{Q_\alpha, \psi_\beta\}^{\mathrm{Op}} = \left(\frac{\delta}{\delta Y^\beta}\partial_{\epsilon^\alpha}\Gamma_{\mathrm{eff}}\right)^{\mathrm{Op}} \equiv \delta_\alpha \psi^{\mathrm{Op}}_\beta\,. \tag{2.58}$$

Analogously for the conjugate quantities.

As a consequence of (2.57), (2.58) and the translations one derives the validity of the algebra

$$\begin{aligned}\left[\{\mathrm{i}Q_\alpha, \mathrm{i}\bar{Q}_{\dot\alpha}\}, \phi\right] &= \{\mathrm{i}Q_\alpha, [\mathrm{i}\bar{Q}_{\dot\alpha}, \phi]\} + \{\mathrm{i}\bar{Q}_{\dot\alpha}, [\mathrm{i}Q_\alpha, \phi]\} \\ &= \{\mathrm{i}\bar{Q}_{\dot\alpha}, \psi_\alpha\} = \frac{\delta}{\delta Y^\alpha}\partial_{\bar\varepsilon^{\dot\alpha}}\Gamma_{\mathrm{eff}} = 2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\phi \\ &= -2\sigma^\mu_{\alpha\dot\alpha}[P_\mu, \phi]\end{aligned} \tag{2.59}$$

Similarly,

$$\begin{aligned}\left[\{\mathrm{i}Q_\alpha, \mathrm{i}\bar{Q}_{\dot\alpha}\}, \psi_\beta\right] &= \left[\mathrm{i}Q_\alpha, \frac{\delta}{\delta Y^\beta}\partial_{\bar\varepsilon^{\dot\alpha}}\Gamma_{\mathrm{eff}}\right] + \left[\mathrm{i}\bar{Q}_{\dot\alpha}, \frac{\delta}{\delta Y^\beta}\partial_{\varepsilon^\alpha}\Gamma_{\mathrm{eff}}\right] \\ &= -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\psi_\beta\,,\end{aligned} \tag{2.60}$$

where the second equality is established by differentiating (2.44) w.r.t. ε^α, $\bar\varepsilon^{\dot\alpha}$, Y^β and performing the LSZ reduction. Thus we have

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu\,. \tag{2.61}$$

The current conservation implies that Q_α, $\bar{Q}_{\dot\alpha}$ act as symmetry operators on the Hilbert space of the theory, with the transformation of field variables given by (2.57), (2.58). This field transformation law is non-linear (for ψ), but it is local! And in this sense one can say that the transformations as given on the functionals (2.43) resemble the operator law here and hint indeed correctly to a local symmetry. We obtain a closed algebra here because the field operators obey the equations of motion.

This concludes our discussion of the Wess-Zumino model.

3 SQED in the Wess-Zumino gauge

The susy and gauge invariant action of SQED in the Wess-Zumino gauge reads as follows

$$\begin{aligned}\Gamma_{\rm SQED} = & -\tfrac{1}{4}\int F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\int \bar\gamma\,(\mathrm{i}\gamma^\mu)\,\partial_\mu\gamma \\ & + \int |D_\mu\phi_L|^2 + \int |D_\mu\phi_R^\dagger|^2 + \bar\Psi\,(\mathrm{i}\gamma^\mu)\,D_\mu\Psi \\ & - \sqrt{2}eQ_L\int \Big(\bar\Psi P_R\gamma\phi_L - \bar\Psi P_L\gamma\phi_R^\dagger + \phi_L^\dagger\bar\gamma P_L\Psi - \phi_R\bar\gamma P_R\Psi\Big) \\ & - \tfrac{1}{2}\int \big(eQ_L|\phi_L|^2 + eQ_R|\phi_R|^2\big)^2 \\ & - m\int \bar\Psi\Psi - m^2\int \big(|\phi_L|^2 + |\phi_R|^2\big)\end{aligned} \tag{3.1}$$

where

$$D_\mu \equiv \partial_\mu + \mathrm{i}eQA_\mu \tag{3.2}$$
$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3.3}$$

and $(A^\mu, \lambda^\alpha, \bar\lambda_{\dot\alpha})$ denote the photon and Weyl-photino resp., whereas (ψ_L^α, ϕ_L) and (ψ_R^α, ϕ_R) refer to two chiral multiplets with charges $Q_L = -1$, $Q_R = +1$. The electron Dirac spinor and the photino Majorana spinor are given by

$$\Psi = \begin{pmatrix}\psi_L^\alpha \\ \bar\psi_R^{\dot\alpha}\end{pmatrix}, \qquad \gamma = \begin{pmatrix}-\mathrm{i}\lambda_\alpha \\ \mathrm{i}\bar\lambda^{\dot\alpha}\end{pmatrix}. \tag{3.4}$$

In this gauge model susy transformations are non-linear not only because the auxiliary fields have been eliminated but also because longitudinal components of the vector superfield are being transformed away. This causes an additional problem: every susy transformation has to be followed by a field dependent gauge transformation such that one stays in the Wess-Zumino gauge. Hence only gauge invariant terms can at the same time be supersymmetric; a gauge fixing term can never be susy invariant. The task is therefore to construct the model and then physical (i.e. gauge invariant) quantities "modulo gauge fixing".

3.1 BRS transformations, ST identity

The problems that susy transformations are non-linear, close only on-shell and that gauge fixing is not supersymmetric have all been overcome by going to a BRS formulation of all transformations and introducing suitable external fields, in particular also terms in $\Gamma_{\rm eff}$

which are bilinear in the latter [5, 6, 4]. These generalized BRS transformations have the form

$$\mathbf{s}\,A_\mu = \partial_\mu c + \mathrm{i}\varepsilon\sigma_\mu\bar\lambda - \mathrm{i}\lambda\sigma_\mu\bar\varepsilon - \mathrm{i}\omega^\nu\partial_\nu A_\mu\,, \tag{3.5a}$$

$$\mathbf{s}\,\lambda^\alpha = \tfrac{\mathrm{i}}{2}(\varepsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma} - \mathrm{i}\varepsilon^\alpha\, eQ_L(|\phi_L|^2 - |\phi_R|^2) - \mathrm{i}\omega^\nu\partial_\nu\lambda^\alpha\,, \tag{3.5b}$$

$$\mathbf{s}\,\bar\lambda_{\dot\alpha} = -\tfrac{\mathrm{i}}{2}(\bar\varepsilon\bar\sigma^{\rho\sigma})_{\dot\alpha} F_{\rho\sigma} - \mathrm{i}\bar\varepsilon_{\dot\alpha}\, eQ_L(|\phi_L|^2 - |\phi_R|^2) - \mathrm{i}\omega^\nu\partial_\nu\bar\lambda_{\dot\alpha}\,, \tag{3.5c}$$

$$\mathbf{s}\,\phi_L = -\mathrm{i}eQ_L c\,\phi_L + \sqrt{2}\,\varepsilon\psi_L - \mathrm{i}\omega^\nu\partial_\nu\phi_L\,, \tag{3.5d}$$

$$\mathbf{s}\,\phi_L^\dagger = \mathrm{i}eQ_L c\,\phi_L^\dagger + \sqrt{2}\,\bar\psi_L\bar\varepsilon - \mathrm{i}\omega^\nu\partial_\nu\phi_L^\dagger\,, \tag{3.5e}$$

$$\mathbf{s}\,\psi_L^\alpha = -\mathrm{i}eQ_L c\,\psi_L^\alpha - \sqrt{2}\,\varepsilon^\alpha\, m\phi_R^\dagger - \sqrt{2}\,\mathrm{i}(\bar\varepsilon\bar\sigma^\mu)^\alpha D_\mu\phi_L - \mathrm{i}\omega^\nu\partial_\nu\psi_L^\alpha\,, \tag{3.5f}$$

$$\mathbf{s}\,\bar\psi_{L\dot\alpha} = \mathrm{i}eQ_L c\,\bar\psi_{L\dot\alpha} + \sqrt{2}\,\bar\varepsilon_{\dot\alpha}\, m\phi_R + \sqrt{2}\,\mathrm{i}(\varepsilon\sigma^\mu)_{\dot\alpha}(D_\mu\phi_L)^\dagger - \mathrm{i}\omega^\nu\partial_\nu\bar\psi_{L\dot\alpha}\,, \tag{3.5g}$$

$$\mathbf{s}\,c = 2\mathrm{i}\varepsilon\sigma^\nu\bar\varepsilon A_\nu - \mathrm{i}\omega^\nu\partial_\nu c\,, \tag{3.5h}$$

$$\mathbf{s}\,\varepsilon^\alpha = 0\,, \tag{3.5i}$$

$$\mathbf{s}\,\bar\varepsilon^{\dot\alpha} = 0\,, \tag{3.5j}$$

$$\mathbf{s}\,\omega^\nu = 2\varepsilon\sigma^\nu\bar\varepsilon\,, \tag{3.5k}$$

$$\mathbf{s}\,\bar c = B - \mathrm{i}\omega^\nu\partial_\nu\bar c\,, \tag{3.5l}$$

$$\mathbf{s}\,B = 2\mathrm{i}\varepsilon\sigma^\nu\bar\varepsilon\partial_\nu\bar c - \mathrm{i}\omega^\nu\partial_\nu B \tag{3.5m}$$

A suitable form of gauge fixing turns out to be

$$\Gamma_{\text{g.f.}} = \int \mathbf{s}\,\left(\bar c\partial A + \tfrac{\xi}{2}\bar c B\right) \tag{3.6}$$

$$= \int \left(B\partial A + \tfrac{\xi}{2}B^2 - \bar c\Box c - \bar c\partial^\mu\left(\mathrm{i}\varepsilon\sigma_\mu\bar\lambda - \mathrm{i}\lambda\sigma_\mu\bar\varepsilon\right) + \xi\mathrm{i}\varepsilon\sigma^\nu\bar\varepsilon\partial_\nu\bar c\bar c\right) \tag{3.7}$$

The non-linear transformations will be defined via their coupling to external fields

$$\Gamma_{\text{ext}} = \int \left(Y_\lambda^\alpha\mathbf{s}\,\lambda_\alpha + Y_{\bar\lambda\dot\alpha}\mathbf{s}\,\bar\lambda^{\dot\alpha} + Y_{\phi_L}\mathbf{s}\,\phi_L + Y_{\phi_L^\dagger}\mathbf{s}\,\phi_L^\dagger + Y_{\psi_L}^\alpha\mathbf{s}\,\psi_{L\alpha} + Y_{\bar\psi_L\dot\alpha}\mathbf{s}\,\bar\psi_L^{\dot\alpha} + (L\to R)\right) \tag{3.8}$$

complemented by well specified correction terms in higher orders.

The contributions which are bilinear in the external fields have the form

$$\Gamma_{\text{bil}} = -\int \left((Y_\lambda\varepsilon)(\bar\varepsilon Y_{\bar\lambda}) + 2(Y_{\psi_L}\varepsilon)(\bar\varepsilon Y_{\bar\psi_L}) + 2(Y_{\psi_R}\varepsilon)(\bar\varepsilon Y_{\bar\psi_R})\right)\,. \tag{3.9}$$

The classical action

$$\Gamma_{\text{cl}} = \Gamma_{\text{SQED}} + \Gamma_{\text{g.f.}} + \Gamma_{\text{ext}} + \Gamma_{\text{bil}} \tag{3.10}$$

satisfies then a Slavnov-Taylor identity which can be extended [4] to all orders in the loop

expansion for the vertex functional Γ,

$$
\begin{aligned}
\mathscr{S}(\Gamma) \equiv \int \mathrm{d}^4x \Big(& \mathbf{s}\, A^\mu \frac{\delta\Gamma}{\delta A^\mu} + \mathbf{s}\, c \frac{\delta\Gamma}{\delta c} + \mathbf{s}\, \bar{c} \frac{\delta\Gamma}{\delta \bar{c}} + \mathbf{s}\, B \frac{\delta\Gamma}{\delta B} \\
& + \frac{\delta\Gamma}{\delta Y_{\lambda\alpha}} \frac{\delta\Gamma}{\delta \lambda^\alpha} + \frac{\delta\Gamma}{\delta Y^{\dot\alpha}_{\bar\lambda}} \frac{\delta\Gamma}{\delta \bar\lambda_{\dot\alpha}} \\
& + \frac{\delta\Gamma}{\delta Y_{\phi_L}} \frac{\delta\Gamma}{\delta \phi_L} + \frac{\delta\Gamma}{\delta Y_{\phi_L^\dagger}} \frac{\delta\Gamma}{\delta \phi_L^\dagger} + \frac{\delta\Gamma}{\delta Y_{\psi_L\alpha}} \frac{\delta\Gamma}{\delta \psi_L^\alpha} + \frac{\delta\Gamma}{\delta Y^{\dot\alpha}_{\bar\psi_L}} \frac{\delta\Gamma}{\delta \bar\psi_{L\dot\alpha}} + (L \to R) \Big) \\
& + \mathbf{s}\, \varepsilon^\alpha \frac{\partial\Gamma}{\partial \varepsilon^\alpha} + \mathbf{s}\, \bar\varepsilon_{\dot\alpha} \frac{\partial\Gamma}{\partial \bar\varepsilon_{\dot\alpha}} + \mathbf{s}\, \omega^\mu \frac{\partial\Gamma}{\partial \omega^\mu} && (3.11) \\
\equiv & \int \left(\mathbf{s}\, \phi'_i \frac{\delta\Gamma}{\delta \phi'_i} + \frac{\delta\Gamma}{\delta Y_i} \frac{\delta\Gamma}{\delta \phi_i} \right) && (3.12) \\
= & \; 0 && (3.13)
\end{aligned}
$$

(ϕ': all linearly transforming field and the ghosts; ϕ: all non-linearly transforming fields). Part of the hypotheses is the gauge fixing as above (this can be established to all orders) and the ghost dependence (s. [4] for details).

An important calculational tool is given by the linearized ST operator

$$
\mathscr{S}_\Gamma \equiv \int \left(\mathbf{s}\, \phi'_i \frac{\delta}{\delta \phi'_l} + \frac{\delta\Gamma}{\delta Y_i} \frac{\delta}{\delta \phi_i} + \frac{\delta\Gamma}{\delta \phi_i} \frac{\delta}{\delta Y_i} \right) \tag{3.14}
$$

which satisfies

$$
\mathscr{S}_\Gamma \, \mathscr{S}(\Gamma) = 0 \tag{3.15}
$$

provided

$$
\mathscr{S}_\Gamma^2 A_\mu = 0\,. \tag{3.16}
$$

(The latter relation is true for the final vertex functional.)

As a consequence of gauge fixing, ghost equations and (3.13) it has been shown in [4] that the following WI's hold

$$
\partial^\mu \frac{\delta\Gamma}{\delta A^\mu} = -\mathrm{i} e w_{\mathrm{em}} \Gamma - \Box B + O(\omega)\ , \tag{3.17}
$$

$$
\begin{aligned}
w_{\mathrm{em}} = Q_L \Big(& \phi_L \frac{\delta}{\delta \phi_L} - Y_{\phi_L} \frac{\delta}{\delta Y_{\phi_L}} + \psi_L \frac{\delta}{\delta \psi_L} - Y_{\psi_L} \frac{\delta}{\delta Y_{\psi_L}} \\
& - \phi_L^\dagger \frac{\delta}{\delta \phi_L^\dagger} + Y_{\phi_L^\dagger} \frac{\delta}{\delta Y_{\phi_L^\dagger}} - \bar\psi_L \frac{\delta}{\delta \bar\psi_L} - Y_{\bar\psi_L} \frac{\delta}{\delta Y_{\bar\psi_L}} \Big) \\
& + (L \to R)
\end{aligned} \tag{3.18}
$$

and

$$
\int \left(\partial_\mu \phi'_i \frac{\delta\Gamma}{\delta \phi'_i} + \partial_\mu \phi_i \frac{\delta\Gamma}{\delta \phi_i} + \partial_\mu Y_i \frac{\delta\Gamma}{\delta Y_i} \right) = 0\,. \tag{3.19}
$$

(3.17) expresses the local gauge invariance of Γ, whereas (3.19) says that Γ is translation invariant.

3.2 Susy on the vertex like functionals

Like in the Wess-Zumino model (section 2.2) we follow [5, 6] and introduce with the help of the operator

$$\mathscr{N} \equiv \varepsilon \frac{\partial}{\partial \varepsilon} + \bar{\varepsilon} \frac{\partial}{\partial \bar{\varepsilon}} + \omega^\mu \frac{\partial}{\partial \omega^\mu} \tag{3.20}$$

a "filtration". $\mathscr{S}_\Gamma$ will be expanded according to the number of constant ghosts:

$$\mathscr{S}_\Gamma = \sum_{n \geq 0} \mathscr{S}_\Gamma^{(n)} \tag{3.21}$$

where

$$[\mathscr{N}, \mathscr{S}_\Gamma^{(n)}] = n \mathscr{S}_\Gamma^{(n)} . \tag{3.22}$$

We have

$$\mathscr{S}(\Gamma) = \mathscr{S}(\Gamma^{(0)}) + \mathscr{S}_{\Gamma^{(0)}} \Gamma^{(1)} + O(\epsilon^2) . \tag{3.23}$$

Like above the nilpotency (3.15) of $\mathscr{S}_\Gamma$ leads to the consequence

$$(\mathscr{S}_\Gamma^{(0)})^2 = 0 \tag{3.24a}$$

$$\mathscr{S}_\Gamma^{(0)} \mathscr{S}_\Gamma^{(1)} + \mathscr{S}_\Gamma^{(1)} \mathscr{S}_\Gamma^{(0)} = 0 \tag{3.24b}$$

$$\mathscr{S}_\Gamma^{(0)} \mathscr{S}_\Gamma^{(2)} + \mathscr{S}_\Gamma^{(1)} \mathscr{S}_\Gamma^{(1)} + \mathscr{S}_\Gamma^{(2)} \mathscr{S}_\Gamma^{(0)} = 0 \tag{3.24c}$$

But it is to be noted that the sector with ghost number 0 is now the one of *ordinary* BRS invariance. Hence $\mathscr{S}_\Gamma^{(0)}$ is the ordinary BRS variation of functionals and (3.24a) expresses its nilpotency.

Decomposing $\mathscr{S}_\Gamma^{(1)}$ again according to

$$\mathscr{S}_\Gamma^{(1)} = \varepsilon^\alpha W_\alpha + \bar{\varepsilon}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + \omega^\mu W_\mu + \varepsilon \sigma^\nu \bar{\varepsilon} \frac{\delta}{\delta \omega^\nu} \tag{3.25}$$

and inserting into (3.24c) one finds

$$\{W_\alpha, \bar{W}_{\dot{\alpha}}\} \sim 2 \sigma^\mu_{\alpha\dot{\alpha}} W_\mu \tag{3.26}$$

$$\{W_\alpha, W_\beta\} \sim 0 \sim \{\bar{W}_{\dot{\alpha}}, \bar{W}_{\dot{\beta}}\} \tag{3.27}$$

i.e. the supersymmetry algebra on functionals $\mathscr{F}(\phi)$ which obey

$$\mathscr{S}_\Gamma^{(0)} \mathscr{F} = 0 \tag{3.28}$$

and where $\sim$ means up to $\mathscr{S}_\Gamma^{(0)}$-variations. On functionals which are not invariant under ordinary BRS transformations the well-known susy algebra is realized only modulo BRS variations. Hence one can interprete W_α and $\bar{W}_{\dot{\alpha}}$ as giving the supersymmetry variation of an arbitrary functional.

The simplest ones are the fields themselves and in [4] several examples have been calculated in the one-loop approximation with the outcome that e.g. $\psi(x)$ transforms non-locally under W_α. The closer analysis of this result is the subject of the subsequent analysis in terms of general Green functions and then of operators.

3.3 Symmetries on quantum fields

For the case of space-time translations there is no difference to the Wess-Zumino model. The WI (3.19) can be turned into a local one, formulated on the general Green functions Z and and translated into operator relations analogous to (2.52).

Similarly we rewrite the gauge WI (3.17) as an eqn. on Z

$$-\mathrm{i}\partial^\mu J_\mu^{\mathrm{em}} \cdot Z = -ew_{\mathrm{em}}(j)\, Z + \mathrm{i}\Box\frac{\delta Z}{\delta j_B} + o(\omega) \tag{3.29}$$

$$0 = \int \left(\partial_\mu j_{\phi_i'}\frac{\delta}{\delta j_{\phi_i'}} + \partial_\mu j_{\phi_i}\frac{\delta}{\delta j_{\phi_i}} + \partial_\mu Y_i\frac{\delta}{\delta Y_i}\right) Z\,. \tag{3.30}$$

and define a conserved electromagnetic current insertion:

$$ew_{\mathrm{em}}(j)Z|_{Y=0,\omega=0} = \mathrm{i}\Box\frac{\delta Z}{\delta j_B} + \partial^\mu J_\mu \cdot Z = \mathrm{i}\partial^\mu j_\mu^{\mathrm{em}} \cdot Z \tag{3.31}$$

We find first of all that the corresponding operator is conserved

$$0 = (\partial^\mu j_\mu^{\mathrm{em}})^{\mathrm{Op}}\,. \tag{3.32}$$

Differentiating once w.r.t. the source j_{ϕ_L} and performing LSZ reduction we obtain

$$-\mathrm{i}eQ_L\delta(y-x)\phi_L^{\mathrm{Op}}(y) = \mathrm{i}\partial^\mu \mathrm{T}\left(j_\mu^{\mathrm{em}}(y)\phi_L(x)\right)^{\mathrm{Op}}\,. \tag{3.33}$$

Integration yields the transformation for the field operator

$$-\mathrm{i}eQ_L\phi^{\mathrm{Op}}(x) = \mathrm{i}[Q^{\mathrm{em}}, \phi_L(x)]^{\mathrm{Op}} \tag{3.34}$$

where

$$Q^{\mathrm{em}} \equiv \int \mathrm{d}^3y\, j_0^{\mathrm{em}}(y)\,, \tag{3.35}$$

similarly for all other field operators. In particular

$$\mathrm{i}[Q^{\mathrm{em}}, A_\mu(x)]^{\mathrm{Op}} = 0\,, \tag{3.36}$$

the field A_μ is not charged [9].

Aiming now at the BRS charge we derive the first consequence from the ST identity directly. (3.13) is rewritten on Z

$$\begin{aligned}\mathscr{S}(Z) \equiv \int \mathrm{d}^4x \Bigg[&\mathrm{i}j_{A\mu}\left(\partial_\mu\frac{\delta Z}{\delta j_c} + \mathrm{i}\varepsilon^\alpha\sigma_{\mu\alpha\dot\alpha}\frac{\delta Z}{\delta\bar\eta_{\lambda\dot\alpha}} - \mathrm{i}\omega^\nu\partial_\nu\frac{\delta Z}{\delta j_{A\mu}}\right) - \mathrm{i}j_c\left(2\mathrm{i}\varepsilon\sigma^\nu\bar\varepsilon\frac{\delta Z}{\delta j_A^\nu} - \mathrm{i}\omega^\nu\partial_\nu\frac{\delta Z}{\delta j_c}\right)\\ &- \mathrm{i}j_{\bar c}\left(\frac{\delta Z}{\delta j_B} - \mathrm{i}\omega^\nu\partial_\nu\frac{\delta Z}{\delta j_{\bar c}}\right) + \mathrm{i}j_B\left(2\mathrm{i}\varepsilon\sigma^\nu\bar\varepsilon\partial_\nu\frac{\delta Z}{\delta j_{\bar c}} - \mathrm{i}\omega^\nu\partial_\nu\frac{\delta Z}{\delta j_B}\right) - \mathrm{i}\eta_\lambda^\alpha\frac{\delta Z}{\delta Y_\lambda^\alpha} - \mathrm{i}\bar\eta_{\lambda\dot\alpha}\frac{\delta Z}{\delta\bar Y_{\lambda\dot\alpha}}\\ &+ \mathrm{i}j_{\phi_L}\frac{\delta Z}{\delta Y_{\phi_L}} + \mathrm{i}\bar j_{\phi_L}\frac{\delta Z}{\delta\bar Y_{\phi_L}} - \mathrm{i}\eta_{\psi_L}^\alpha\frac{\delta Z}{\delta Y_{\psi_L}^\alpha} - \mathrm{i}\bar\eta_{\psi_L\dot\alpha}\frac{\delta Z}{\delta\bar Y_{\bar\psi_L\dot\alpha}} + (L \longrightarrow R)\Bigg]\\ &- 2\mathrm{i}\varepsilon\sigma^\mu\bar\varepsilon\frac{\partial Z}{\partial\omega^\mu} = 0\end{aligned} \tag{3.37}$$

If a local ST identity is desired we have to permit the ghosts ε, $\bar{\varepsilon}$, ω^μ to become *local*. The local ST identity will then look as follows

$$
\begin{aligned}
\mathscr{S}_{\mathrm{loc}} Z \equiv\ & \mathrm{i} j_{A\mu}\left(\partial_\mu \frac{\delta Z}{\delta j_c}+\mathrm{i}\varepsilon^\alpha \sigma_{\mu\alpha\dot{\alpha}} \frac{\delta Z}{\delta \bar{\eta}_{\lambda\dot{\alpha}}}-\mathrm{i}\omega^\nu \partial_\nu \frac{\delta Z}{\delta j_{A\mu}}\right)-\mathrm{i} j_c\left(2\mathrm{i}\varepsilon\sigma^\nu \bar{\varepsilon} \frac{\delta Z}{\delta j_A^\nu}-\mathrm{i}\omega^\nu \partial_\nu \frac{\delta Z}{\delta j_c}\right) \\
& -\mathrm{i} j_{\bar{c}}\left(\frac{\delta Z}{\delta j_B}-\mathrm{i}\omega^\nu \partial_\nu \frac{\delta Z}{\delta j_{\bar{c}}}\right)+\mathrm{i} j_B\left(2\mathrm{i}\varepsilon\sigma^\nu \bar{\varepsilon} \partial_\nu \frac{\delta Z}{\delta j_{\bar{c}}}-\mathrm{i}\omega^\nu \partial_\nu \frac{\delta Z}{\delta j_B}\right)-\mathrm{i}\eta_\lambda^\alpha \frac{\delta Z}{\delta Y_\lambda^\alpha}-\mathrm{i}\bar{\eta}_{\lambda\dot{\alpha}} \frac{\delta Z}{\delta \bar{Y}_{\lambda\dot{\alpha}}} \\
& +\mathrm{i} j_{\phi_L} \frac{\delta Z}{\delta Y_{\phi_L}}+\mathrm{i}\bar{j}_{\phi_L} \frac{\delta Z}{\delta \bar{Y}_{\phi_L}}-\mathrm{i}\eta_{\psi_L}^\alpha \frac{\delta Z}{\delta Y_{\psi_L}^\alpha}-\mathrm{i}\bar{\eta}_{\psi_L\dot{\alpha}} \frac{\delta Z}{\delta \bar{Y}_{\bar{\psi}_L\dot{\alpha}}}+(L \longrightarrow R) \\
& -2\mathrm{i}\varepsilon\sigma^\mu \bar{\varepsilon} \frac{\delta Z}{\delta \omega^\mu} \\
=\ & \left[\partial^\mu J_\mu+\partial^\mu \varepsilon^\alpha K_{\mu\alpha}+\partial^\mu \bar{\varepsilon}^{\dot{\alpha}} \bar{K}_{\mu\dot{\alpha}}+\partial^\mu \omega^\nu K_{\mu\nu}\right] \cdot Z
\end{aligned}
\tag{3.38}
$$

Here J_μ depends on the ghosts ε, $\bar{\varepsilon}$, ω^μ and for constant ghosts integration will indeed yield zero on the r.h.s. At $\varepsilon=\bar{\varepsilon}=\omega=0$ we are obviously dealing with a local version of the ordinary ST identity, i.e. $\partial^\mu J_\mu|_{\varepsilon=\bar{\varepsilon}=\omega=0}$ will correspond to the divergence of the BRS current. Reduction leads to the operator equation

$$
0=\left(\partial^\mu J_\mu\right)^{\mathrm{Op}}\Big|_{\varepsilon=\bar{\varepsilon}=\omega=0} \equiv\left(\partial^\mu J_\mu^{\mathrm{BRS}}\right)^{\mathrm{Op}}
\tag{3.39}
$$

The conserved BRS current leads as usual to a time independent charge via the definition

$$
Q^{\mathrm{BRS}}=-\int \mathrm{d}^3 x\, J_0^{\mathrm{BRS}} .
\tag{3.40}
$$

Since (3.39) holds on the complete Fock space Q^{BRS} commutes with the S-operator on Fock space [10].

The transformation law for a field operator of type ϕ follows from the operator eqn.

$$
-\delta(y-x) \frac{\delta \Gamma_{\mathrm{eff}}^{\mathrm{Op}}}{\delta Y(y)}=\mathrm{i}\, \mathrm{T}\left(\partial^\mu J_\mu(y) \phi(x)\right)^{\mathrm{Op}}\Big|_{\epsilon=\bar{\epsilon}=0}
\tag{3.41}
$$

by integrating over space,$\mathrm{d}^3 y$, and time in the interval $(x_0-\mu, x_0+\mu)$ with $\mu \rightarrow 0$ and leads to

$$
\left.\left(\frac{\delta \Gamma_{\mathrm{eff}}}{\delta Y_\phi(x)}\right)^{\mathrm{Op}}\right|_{\varepsilon=\bar{\varepsilon}=\omega=0}=\mathrm{i}\left[Q^{\mathrm{BRS}}, \phi(x)\right]^{\mathrm{Op}} .
\tag{3.42}
$$

$\left(\frac{\delta \Gamma_{\mathrm{eff}}}{\delta Y_\phi(x)}\right)^{\mathrm{Op}}$ starts with the non-linear terms given by the tree approximation and listed in (3.5); higher order corrections appear as they contribute to Γ_{eff}.

For a linearly transforming field, type ϕ', the operator law is the one of the classical approximation

$$
\mathbf{s}\, \phi'^{\mathrm{Op}}(x)=\mathrm{i}\left[Q^{\mathrm{BRS}}, \phi'(x)\right]^{\mathrm{Op}} .
\tag{3.43}
$$

The charge $Q^{\rm BRS}$ is nilpotent.

As is clear from the discussion in section 2.3 information on supersymmetry follows from (3.38) by differentiating once with respect to a local susy ghost ($\varepsilon^\alpha(z)$ or analogously $\bar\varepsilon_{\dot\alpha}(z)$). Doing so and performing LSZ reduction we obtain the operator equation

$$0 = \frac{\delta}{\delta\varepsilon^\alpha(z)}\partial^\mu_x J^{\rm Op}_\mu(x) + {\rm i}\,{\rm T}\left(\partial^\mu J_\mu(x)\,\frac{\delta\Gamma_{\rm eff}}{\delta\varepsilon^\alpha(z)}\right)^{\rm Op} + \partial^\mu_x\delta(x-z)\,K^{\rm Op}_{\mu\alpha}(x)\,. \tag{3.44}$$

Integration over z yields

$$0 = \partial^\mu_x\,\partial_{\varepsilon^\alpha}J^{\rm Op}_\mu(x) + {\rm i}\int {\rm d}^4z\,{\rm T}\left(\partial^\mu J_\mu(x)\,\frac{\delta\Gamma_{\rm eff}}{\delta\varepsilon^\alpha(z)}\right)^{\rm Op}\,. \tag{3.45}$$

Hence there is a candidate for a susy current, $\partial_{\varepsilon^\alpha}J^{\rm Op}_\mu(x)$, but it is not conserved. Defining a charge by

$$Q_\alpha(t) \equiv -\int {\rm d}^3x\,\partial_{\varepsilon^\alpha}J^{\rm Op}_0(x) \tag{3.46}$$

it will depend on t. Integrating (3.45) over all of x-space leads to

$$0 = \int {\rm d}t\,\partial_t Q_\alpha(t) - {\rm i}\left[Q^{\rm BRS},\partial_{\varepsilon^\alpha}\Gamma_{\rm eff}\right]^{\rm Op}\,. \tag{3.47}$$

Taking the time integral for asymptotic times $t=\pm\infty$ and identifying there the charges we can write

$$Q^{\rm out}_\alpha - Q^{\rm in}_\alpha = {\rm i}\left[Q^{\rm BRS},\partial_{\varepsilon^\alpha}\Gamma_{\rm eff}\right]^{\rm Op}\,. \tag{3.48}$$

Since $Q^{\rm out}_\alpha$ develops out of $Q^{\rm in}_\alpha$ via the time evolution operator S, the scattering operator,

$$Q^{\rm out}_\alpha = SQ^{\rm in}_\alpha S^\dagger\,, \tag{3.49}$$

(3.48) implies

$$[Q^{\rm in}_\alpha, S] = -{\rm i}[Q^{\rm BRS},\partial_{\varepsilon^\alpha}\Gamma_{\rm eff}\cdot S]\,. \tag{3.50}$$

Here we have used that $Q^{\rm BRS}$ and S commute. The interpretation of this result is clear: the charge $Q^{\rm in}_\alpha$ which may be taken to be the generator of susy transformations on the free in-states does not commute with the S-operator, the reason being the ε-dependence of $\Gamma_{\rm eff}$. Looking at (3.7) this arises from the gauge fixing term whose ϵ-dependent part is as a consequence of the ghost eqn. not renormalized. Matrix elements between physical states however yield a vanishing r.h.s. in (3.50), hence there $Q^{\rm in}_\alpha$ is a conserved charge.

In the derivation of analogous results for field transformations we concentrate on non-linearly transforming ones. Differentiating (3.38) w.r.t. ε and a source for ϕ we obtain

after LSZ reduction the operator relation[4]

$$
\begin{aligned}
-\delta(y-x)\,\frac{\delta^2\Gamma_{\text{eff}}}{\delta Y(y)\,\delta\varepsilon^\alpha(z)} &- \mathrm{i}\,\delta(y-x)\mathrm{T}\left(\frac{\delta\Gamma_{\text{eff}}}{\delta Y(y)}\frac{\delta\Gamma_{\text{eff}}}{\delta\varepsilon^\alpha(z)}\right) \\
&= \mathrm{i}\,\mathrm{T}\left(\frac{\delta}{\delta\varepsilon^\alpha(z)}\partial^\mu J_\mu(y)\phi(x)\right) - \mathrm{T}\left(\partial^\mu J_\mu(y)\frac{\delta\Gamma_{\text{eff}}}{\delta\varepsilon^\alpha(z)}\phi(x)\right) \\
&\quad + \mathrm{i}\,\partial_y^\mu\delta(y-z)\mathrm{T}\left(K_{\mu\alpha}(y)\phi(x)\right)\,. \qquad (3.51)
\end{aligned}
$$

Unfortunately one cannot straightforwardly integrate (3.51) and conclude how a field ϕ transforms under susy because in the T-product $\mathrm{T}(\partial J\cdot\partial_\epsilon\Gamma_{\text{eff}}\cdot\phi)$ distributional singularities at coninciding points may arise. It is clear that every renormalization scheme leads to a well-defined T-product which is integrable in the sense of distributions but of which type this distribution is has to be found out. We present this analysis in the appendix and quote here only its result. One finds that contributions with $\delta(y-x)$ arise which cancel with the operator product $\mathrm{T}(\delta\Gamma_{\text{eff}}/\delta Y(y)\,\cdot\,\delta\Gamma_{\text{eff}}/\delta\epsilon^\alpha(z))$ on the l.h.s., and in addition a term with a double delta function $\delta(x-y)\delta(y-z)$ which contributes to the susy variation of the photino field λ only. Altogether we arrive at

$$
\frac{\delta}{\delta Y_\phi(x)}\partial_{\varepsilon^\alpha}\Gamma_{\text{eff}}^{\text{Op}} = \mathrm{i}\left[Q_\alpha(x^0),\phi(x)\right]^{\text{Op}}\,, \qquad \text{if } \phi\neq\lambda \qquad (3.52)
$$

$$
\frac{\delta}{\delta Y_\lambda^\beta(x)}\partial_{\varepsilon^\alpha}\Gamma_{\text{eff}}^{\text{Op}} + \varepsilon_{\alpha\beta}B^{\text{Op}}(x) = \mathrm{i}\left[Q_\alpha(x^0),\lambda_\beta(x)\right]^{\text{Op}}\,. \qquad (3.53)
$$

Hence the time-dependent susy charge defined in (3.46) ($t\equiv x_0$) generates a local, non-linear transformation on all fields. The fact that this transformation does not correspond to a symmetry on the Fock space, but only on the Hilbert space of the theory is completely encoded in the time dependence of the charge $Q_\alpha(t)$ and in the additional $\varepsilon_{\alpha\beta}B$ term in the susy transformation of λ^α which vanishes between physical states.

Comparing this result with the explicit one-loop calculation in [4] of $\mathscr{S}_\Gamma\psi(x)$ which yielded a non-local expression we can trace the origin of this non-locality: it comes from the operator product $\delta\Gamma_{\text{eff}}/\delta Y(y)\,\cdot\,\delta\Gamma_{\text{eff}}/\delta\epsilon^\alpha(z)$ in the l.h.s. of (3.51) which could be separated in the above analysis and seen to cancel against the $\delta(y-x)$ terms on the r.h.s. By going to the operator level one could isolate the local transformation. The transformation on vertex type functionals given by $\mathscr{S}_\Gamma^{(1)}$ does not distinguish between these different contributions since it fixes in a sense susy transformations only "up to" BRS variations.

The somewhat surprising modification (3.53) of the transformation of λ^α may be understood as follows. We have defined a time-dependent susy charge $Q_\alpha(x^0)$ which generates susy transformations $[Q_\alpha(x^0),\lambda_\beta(x^0,\vec{x})]$ at time x^0. The time dependence of all these operators is determined by the same unitary time evolution operator in Fock space, i.e.

[4]The explicit indication Op has been suppressed

our susy transformation is compatible with this time dependence. However, the equation of motion for λ^α contains contributions from the gauge fixing term which are not supersymmetric and therefore the time dependence of λ^α cannot be compatible with ordinary susy. This shows that $Q_\alpha(x^0)$ cannot generate the standard susy transformation of λ^α. More explicitly, the equation of motion for $\bar\lambda^{\dot\alpha}$ has the form

$$\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\lambda^\alpha = \bar F_{\dot\alpha} + \mathrm{i}\varepsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar c\,, \tag{3.54}$$

where $\bar F_{\dot\alpha}$ contains all interaction terms of the classical action (3.3). This equation is invariant under the combined gauge+susy BRS transformations (3.5), which means to first order in ε:

$$\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\left(\mathbf{s}^{\,\mathrm{susy}}\lambda^\alpha\right) = \mathbf{s}^{\,\mathrm{susy}}\bar F_{\dot\alpha} + \mathrm{i}\varepsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\left(\mathbf{s}^{\,\mathrm{gauge}}\bar c\right)\,, \tag{3.55}$$

where

$$\mathbf{s}^{\,\mathrm{gauge}} = \left(\varepsilon^\alpha\frac{\partial}{\partial\varepsilon^\alpha} + \bar\varepsilon^{\dot\alpha}\frac{\partial}{\partial\bar\varepsilon^{\dot\alpha}}\right)\mathbf{s}\Big|_{\varepsilon=\bar\varepsilon=\omega=0} \tag{3.56}$$

$$\mathbf{s}^{\,\mathrm{susy}} = \mathbf{s}\,\big|_{\varepsilon=\bar\varepsilon=\omega=0}\,. \tag{3.57}$$

Like in the filtration (3.23), supersymmetry holds only up to gauge-BRS transformations, in particular we have

$$\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\left(\mathbf{s}^{\,\mathrm{susy}}\lambda^\alpha\right) \neq \mathbf{s}^{\,\mathrm{susy}}\bar F_{\dot\alpha}\,. \tag{3.58}$$

Since $s^{\mathrm{gauge}}\bar c = B$, (3.55) may be rewritten as

$$\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\left(\mathbf{s}^{\,\mathrm{susy}}\lambda^\alpha - \varepsilon^\alpha B\right) = \mathbf{s}^{\,\mathrm{susy}}\bar F_{\dot\alpha}\,. \tag{3.59}$$

This shows that the modification of the susy transformation law of λ^α ensures compatibility with time evolution.

4 Discussion and Conclusions

Comparing the results of our analysis for the Wess-Zumino model and SQED in the Wess-Zumino gauge we find

- Closely analogous ones: translations and gauge transformations obey simple WIs and can easily be interpreted in terms of operators; susy requires the use of the generalized ST identity expressing the generalized BRS invariance; its non-linear character does not really cause harm;

- clearly different ones: in the Wess-Zumino model susy is not broken although non-linearly realized; its charge exists and is time independent, the transformation is local; in SQED susy is broken by the gauge fixing term and the BRS type formulation only enables one to carry that breaking along in a fashion which permits renormalization and consistent treatment to all orders (the gauge fixing term is a BRS variation); the susy charge is time dependent, but in such a way that the dependence disappears between physical states; remarkably enough there exists still a local operator expression for the field transformations. The susy transformation of the field λ^α is modified by the term $\varepsilon_\alpha B$ which vanishes between physical states.

The comparison with the linear realization of supersymmetry which is possible in these two examples yields qualitative agreement. In the Wess-Zumino model the respective charges (P_μ, Q_α, $\bar{Q}_{\dot\alpha}$) exist and operate on a Hilbert space which is essentially the same as the one generated without auxiliary fields. The latter can be understood as interpolating fields which are useful but not necessary.

In the massive SQED the charges P_μ, Q_α, $\bar{Q}_{\dot\alpha}$ also exist, likewise the "electric" charges because there exist conserved currents which are gauge invariant. All transformations are linear, given by WIs hence the implementation on the Fock space is straightforward. The transformation laws for operators are local. Here, of course, the Fock spaces differ tremendously, but one expects (although a detailed proof seems to be missing) that the Hilbert spaces are equivalent and on them the theories should coincide, in particular the charges P_μ, Q_α, $\bar{Q}_{\dot\alpha}$ and the (true) electric charge Q.

A first comment is in order as far as our rather careless treatment of operators in SQED with massless photon and photino is concerned. With some additional equipment we could have introduced also in the Wess-Zumino gauge a mass term for photon and photino (namely by admitting some susy partners for c, $\bar{c}$ etc.). With an eye on the linear realization we did not do so: our main interest was to clarify the role of the gauge fixing term and the understanding of the breaking of susy which its presence causes in the Wess-Zumino gauge formulation of supersymmetry. The infraredwise existence of operators was not our concern – in this respect our work is formal.

A second comment is appropriate as far as the breaking of susy is concerned. Since it takes place only in Fock space but not in Hilbert space we have the impression that the "collapse of supersymmetry" as described in the paper by Buchholz, Ojima et al. (s. [11]) is still avoided for the physical part of the theory.

A third comment finally refers to the type of operator equations which we have derived. Obviously we worked in perturbation theory and therefore in a very specific representation of the (anti-) commutation relations. Hence these equations cannot be considered as abstract ones, valid for any arbitrary representation[5]. In fact, it seems to be an open problem in which sense they could be "lifted" to have representation independent meaning.

[5]We are very much indebted to H. Grosse for clarifying discussions of this point.

Or stated differently: it is not known which equations valid in perturbation theory hold also beyond perturbation theory. On the other hand it is also obvious that the abstract non-perturbative approach to quantum field theory is not yet able to handle the type of charges dealt with in this paper.

A Singularity structure of some T-products

The starting point for the present analysis is the local ST identity

$$\mathscr{S}_{\rm loc} Z = \left(\partial^\mu J_\mu + \partial^\mu \epsilon^\alpha K_{\mu\alpha} + \partial^\mu \bar\epsilon^{\dot\alpha} K_{\mu\dot\alpha} + \partial^\mu \omega^\nu K_{\mu\nu}\right) \cdot Z \tag{A.1}$$

with with $\mathscr{S}_{\rm loc}$ given by (3.38). Since we are mainly interested in the properties of a non-linearly transforming field, say $\phi(x)$, under superymmetry we differentiate w.r.t. the source for ϕ and $\epsilon^\alpha(z)$. This gives for a general Green function

$$\begin{aligned}
&\delta(y-x)\left(\left\langle \frac{\delta^2\Gamma_{\rm eff}}{\delta\epsilon^\alpha(z)\,\delta Y_\phi(x)}\,X\right\rangle + \left\langle \frac{\delta\Gamma_{\rm eff}}{\delta\epsilon^\alpha(z)}\,\frac{\delta\Gamma_{\rm eff}}{\delta Y_\phi(x)}\,X\right\rangle\right)\\
&+\sum_k \delta(y-x_k)\left(\left\langle \frac{\delta^2\Gamma_{\rm eff}}{\delta\epsilon^\alpha(z)\,\delta Y_{\phi_k}(x_k)}\,\phi(x)\,X_{\check k}\right\rangle + \left\langle \frac{\delta\Gamma_{\rm eff}}{\delta\epsilon^\alpha(z)}\,\frac{\delta\Gamma_{\rm eff}}{\delta Y_{\phi_k}(x_k)}\,X_{\check k}\right\rangle\right)\\
&+\sum_{k'} \delta(y-x_{k'})\left(\left\langle \left(\frac{\delta(\mathbf{s}\,\phi_{k'}(y))}{\delta\epsilon^\alpha(z)}\right)\,\phi(x)\,X_{\check k'}\right\rangle + \left\langle \mathbf{s}\,\phi_{k'}(y)\,\frac{\delta\Gamma_{\rm eff}}{\delta\epsilon^\alpha(z)}\,\phi(x)\,X_{\check k'}\right\rangle\right)\\
&\qquad\qquad = \left\langle \left(\frac{\delta\partial^\mu J_\mu(y)}{\delta\epsilon^\alpha(z)}\Big|_{\epsilon=0} + \delta(y-z)\partial^\mu J_{\mu\alpha}(y)\right)\,\phi(x)\,X\right\rangle\\
&\qquad\qquad\qquad + \left\langle \partial^\mu J_\mu(y)\Big|_{\epsilon=0}\,\frac{\delta\Gamma_{\rm eff}}{\delta\epsilon^\alpha(z)}\,\phi(x)\,X\right\rangle
\end{aligned}\tag{A.2}$$

Here denotes X a general string of fields, $X_{\check k}$ indicates that the field with index k is missing, the same for $X_{\check k'}$. Fields ϕ' transform linearly. $\partial^\mu J_\mu|_{\epsilon=0} = \partial^\mu J_\mu^{\rm BRS}$ is the conserved BRS current.

The problem is now to identify within

$$\begin{aligned}
\delta(y-x)\left(\frac{\delta^2\Gamma_{\rm eff}}{\delta\epsilon(z)\,\delta Y_\phi(y)} + \mathrm{T}\left(\frac{\delta\Gamma_{\rm eff}}{\delta\epsilon(z)}\,\frac{\delta\Gamma_{\rm eff}}{\delta Y_\phi(y)}\right)\right) &= \mathrm{T}\left(\frac{\delta\partial^\mu J_\mu(y)}{\delta\epsilon(z)}\,\phi(x)\right)\\
&\quad + \partial^\mu_y\delta(y-z)\mathrm{T}\left(K_{\mu\alpha}(y)\,\phi(x)\right)\\
&\quad + \mathrm{T}\left(\partial^\mu J_\mu^{\rm BRS}(y)\,\frac{\delta\Gamma_{\rm eff}}{\delta\epsilon(z)}\,\phi(x)\right)
\end{aligned}\tag{A.3}$$

the singularities which form $\delta(y-z)$ or $\delta(z-x)$ or both. These singularities determine the possible contributions from the T-product when it is integrated over y and z. In

particular, we would like to integrate in the order

$$\int\limits_{x_0-\mu}^{x_0+\mu} \mathrm{d}y_0 \int \mathrm{d}^4 z \int \mathrm{d}^3 y \tag{A.4}$$

and take the limit $\mu \to 0$. Clearly, we can get a contribution only from terms containing a factor $\delta(x-y)$. In the triple insertion

$$\left\langle \partial^\mu J_\mu^{\mathrm{BRS}}(y) \, \frac{\delta\Gamma_{\mathrm{eff}}}{\delta\epsilon^\alpha(z)} \, \phi(x) \, X \right\rangle , \tag{A.5}$$

δ-functions may arise when a suitable number of partial derivatives form a wave equation operator and act on the corresponding propagator, e.g.

$$\Box\langle c \, \bar{c}\rangle(y-z) = \mathrm{i}\delta(y-z) \, , \tag{A.6}$$

$$\Box\langle B \, A_\mu\rangle(y-u) = \mathrm{i}\partial_\mu\delta(y-u) \, , \tag{A.7}$$

$$\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\langle\lambda_\beta \, \bar{\lambda}^{\dot{\alpha}}\rangle(y-z) = \varepsilon_{\alpha\beta}\delta(y-z) \, . \tag{A.8}$$

In terms of diagrams, the propagator shrinks then to a point and is replaced by a δ-function.

We first consider the case that $\partial^\mu J_\mu^{\mathrm{BRS}}(y)$ and $\phi(x)$ are directly connected by a line, leading to a contribution with $\delta(x-y)$. Comparing with the WI for ordinary BRS transformations into which one inserts a vertex $\frac{\delta\Gamma_{\mathrm{eff}}}{\delta\epsilon}$ “by hand” it is clear that such contributions cancel against the double insertion

$$\delta(y-x) \left\langle \frac{\delta\Gamma_{\mathrm{eff}}}{\delta\epsilon} \, \frac{\delta\Gamma_{\mathrm{eff}}}{\delta Y_\phi} \, X \right\rangle \tag{A.9}$$

on the l.h.s. of (A.2), including all possible double delta functions $\delta(y-x)\delta(y-z)$.

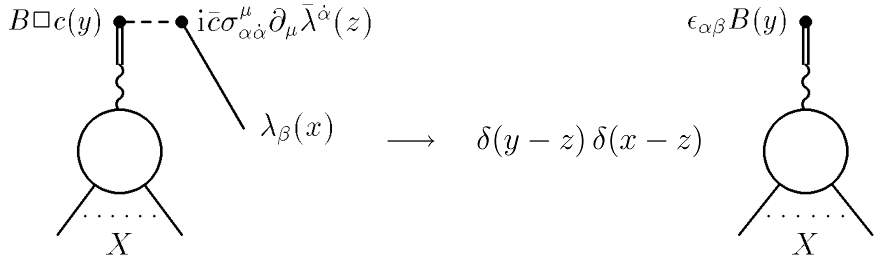
However, there is a second possibility how $\delta(x-y)$ may be generated: If the vertices are connected in the order

$$\phi(x) - - - \frac{\delta\Gamma_{\mathrm{eff}}}{\delta\varepsilon(z)} - - - \partial^\mu J_\mu^{\mathrm{BRS}}(y)$$

we may obtain a double δ function $\delta(x-z)\delta(z-y)$ which is equivalent to $\delta(x-y)\delta(z-y)$ but is not present in the double insertion (A.9). This can only happen for $\phi = \lambda^\alpha$ (or $\bar{\lambda}^{\dot{\alpha}}$), the relevant contributions being

$$\partial^\mu J_\mu^{\mathrm{BRS}} = B\Box c + \dots \tag{A.10}$$

$$\frac{\delta\Gamma_{\mathrm{eff}}}{\delta\varepsilon^\alpha} = \mathrm{i}\bar{c}\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}^{\dot{\alpha}} + \dots \, . \tag{A.11}$$



(A.12)

Here, the lines - - - and represent propagators $\langle c\,\bar{c}\rangle$ resp. $\langle A^\nu\, B\rangle$.

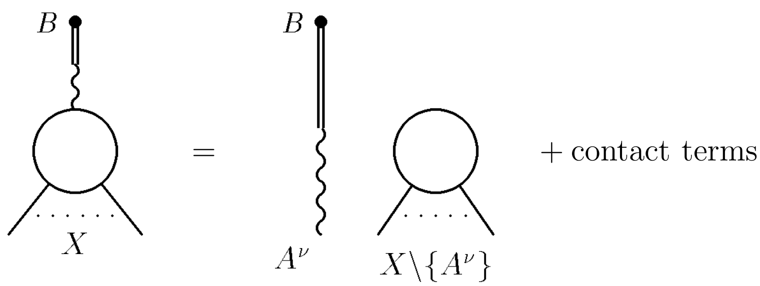
According to (A.6), (A.8), the diagram on the l.h.s. of (A.12) corresponds to a contribution

$$\delta(x-y)\delta(y-z)\langle\varepsilon_\alpha B(x)\,X\rangle\,. \tag{A.13}$$

We will now further investigate this contribution in order to clarify for which field configurations X it is really non-vanishing. From the Ward identity (3.17), it follows immediately that the operator B^{Op} is a free field,

$$\Box B^{\mathrm{Op}} = 0\,. \tag{A.14}$$

Therefore there is only one contribution to the diagram on the right hand side of (A.12), namely



(A.15)

Reduction w.r.t. A^ν yields zero, but reduction w.r.t. B is non-zero. Thus we see that the insertion of $\varepsilon_\alpha B$ (A.13) is indeed nonvanishing, but it is made up entirely of trivial contributions.

Taking into account the above discussions, (A.3) reads

$$\delta(y-x)\frac{\delta^2\Gamma_{\mathrm{eff}}}{\delta\varepsilon\delta Y_\phi} = \mathrm{T}\left(\left(\frac{\delta\partial^\mu J_\mu}{\delta\varepsilon} + \partial^\mu\delta(y-z)K_{\mu\alpha}\right)\,\phi(x)\right) + \text{less singular}\,, \qquad \text{if } \phi\neq\lambda^\alpha \tag{A.16}$$

$$\delta(y-x)\frac{\delta^2\Gamma_{\mathrm{eff}}}{\delta\varepsilon\delta Y^\alpha_\lambda} = \mathrm{T}\left(\left(\frac{\delta\partial^\mu J_\mu}{\delta\varepsilon} + \partial^\mu\delta(y-z)K_{\mu\alpha}\right)\,\lambda^\alpha(x)\right) + \delta(y-x)\delta(y-z)\epsilon^\alpha B(x)$$
$$+ \text{less singular}\,. \tag{A.17}$$

Here, "less singular" stands for terms containing no singularity of the type $\delta(x-y)$. Integration in the order (A.4) yields a local susy variation for all fields,

$$\frac{\delta}{\delta Y_\phi(x)}\partial_{\varepsilon^\alpha}\Gamma^{\mathrm{Op}}_{\mathrm{eff}} = \mathrm{i}\left[Q_\alpha(x^0),\phi(x)\right]^{\mathrm{Op}}\,, \qquad \text{if } \phi\neq\lambda \tag{A.18}$$

$$\frac{\delta}{\delta Y^\beta_\lambda(x)}\partial_{\varepsilon^\alpha}\Gamma^{\mathrm{Op}}_{\mathrm{eff}} + \varepsilon_{\alpha\beta}B(x) = \mathrm{i}\left[Q_\alpha(x^0),\lambda_\beta(x)\right]^{\mathrm{Op}}\,. \tag{A.19}$$

We still note that these expressions are covariant, i.e. renormalization scheme independent, because they are unique: after the double insertion contributions cancelled each other the covariance is guaranteed.

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